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AUTHOR Reckase, Mark D.
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ABSTRACT

The work presented in this paper defined conceptually the concepts of multidimensional discrimination and information, derived mathematical expressions for the concepts for a particular multidimensional item response theory (IRT) model, and applied the concepts to actual test data. Multidimensional discrimination was defined as a function of the slope of the item response surface in the direction specified by the multidimensional difficulty. For the multidimensional extension of the two-parameter logistic (MLPL) model, this definition resulted in a statistic that had the same relationship to multidimensional difficulty that "a" did to "b" for the unidimensional IRT models. This statistic was defined as the square root of the sum of squared a-parameters from the MLPL. Multidimensional information was defined by the same mathematical function as unidimensional information, but the directional derivative was substituted for the standard derivative in the numerator of the information expression. The use of the directional derivative resulted in an information measure that corresponded to a direction in ability space. The American College Testing Program's ACT Assessment Mathematics Usage Test was used to demonstrate the multidimensional discrimination and information. The multidimensional measures of item quality, item precision, and test precision provide tools to gain a better understanding of the measurement process. (PN)

The Discriminating Power of Items that Measure More than One Dimension

Mark D. Reckase

The American College Testing Program

Running head: THE DISCRIMINATING POWER OF ITEMS

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The Discriminating Power of Items that Measure More than One Dimension

Statistical measures of item discrimination are computed for several reasons. The traditional measures, usually the point-biserial or biserial correlations between item scores and total scores, are used as general indicators of item quality, or as screening variables for use in selecting items for a test. During test construction, items are often selected to have a discrimination index that is greater than a specified value, such as .20 or .30. Discrimination indices may also be used to determine whether an item is measuring the same construct as the total score on the test. For the most part, the newer, item response theory (IRT) measures of discrimination and the related concept, information, are used in the same way, but they are also used to specify the precision of measurement provided by an item at different levels of ability along the ability scale.

Both the traditional and IRT measures of the discriminating power of an item are based on the assumption that a test measures a single trait--either that defined by the total score, or by the θ-scale. This paper will generalize the concept of item discrimination to the case where more than one ability is required to determine the correct response to an item. In making this generalization, the conceptual framework supplied by IRT will be

used. This paper will also draw upon the definition of multidimensional item difficulty (MID) developed by Reckase (1985). This definition defines MID as the direction and distance from the origin of the space to the point of steepest slope. Because of the close connection within IRT between item discrimination and item information, a multidimensional extension of the concept of item information will also be presented.

This paper is composed of three parts. First, the concept of discrimination for multidimensional items will be developed in general terms. Second, the general definition will be applied to a particular multidimensional IRT model to determine the mathematical expression for multidimensional discrimination and information for that case. Finally, measures of multidimensional discrimination and information will be computed using item response data from a form of the ACT Mathematics Usage Test and the use of the statistics will be discussed.

Theoretical Framework

The work presented in this paper is based upon the assumption that the interaction between a person and an item can be described by one of a class of well behaved probability functions. These functions relate the probability of a correct response to an item to a person's location in a multidimensional ability space, as indicated by their θ-vector, and the characteristics of the item,

as indicated by a vector of item parameters, σ. That is,

$$P\,(\underline{x}_{ij} = 1) = f\,(\sigma_i, \theta_j)\ , \qquad (1)$$

where $\underline{x}_{ij}$ is the score on Item $\underline{i}$ for Person $\underline{j}$. This function is assumed to be "well behaved" in that for all dimensions in the space, or any combinations of dimensions, as θ_j increases, $\underline{P}\,(\underline{x}_{ij} = 1)$ is nondecreasing. McKinley and Reckase (1982) describe a number of multidimensional item response theory (MIRT) models that are well behaved in the sense described here.

Multidimensional Discrimination

A measure of item discrimination for an item whose performance can be described by Equation 1 will be useful to the extent that it provides the same type of information that is provided by the unidimensional discrimination statistics. That is, the multidimension measure of discrimination (MDISC) should allow items to be compared on a general measure of quality, to be classified as above or below a standard of quality, and to be used as an indicator of strength of relationship of the item performance to dimensions in the ability space. Further, it would be especially convenient if the MDISC statistics were related to the MID statistics in the same way that unidimensional IRT item statistics are related so that some of the interpretive framework that has been developed for the unidimensional case can be

generalized to the multidimensional case.

In unidimensional IRT, the discrimination parameter is related to the slope of the item characteristic curve (ICC) at the point where the slope is steepest, the point of inflection. The point of inflection also defines the difficulty parameter for the items. The most direct generalization of this concept to the multidimensional case would be to relate MDISC to the slope of the surface defined by Equation 1 at the point of steepest slope in the multidimensional space. However, this definition of MDISC has several problems. For some mathematical forms of the function in Equation 1 (e.g., the noncompensatory models), the point of maximum slope occurs when at least one of the θ's approaches infinity. Therefore, the MDISC statistics could not be related to MID in the same way that is true for the unidimensional models. For other classes of models (e.g., the compensatory models), there are an infinite number of points of maximum slope. This fact also causes difficulties when interpreting the MDISC statistic. Therefore, an alternative definition for MDISC will be proposed that uniquely determines a single value for MDISC and that closely relates it to the MID.

The definition proposed in this paper specifies the MDISC as a function of the slope of the item response surface (IRS) defined by Equation 1 at the steepest point in the direction indicated by the MID. Conceptually, determining the MDISC value can be thought

of as requiring several steps. First, a direction from the origin of the space is selected and the point of inflection of the IRS along the line defined by this direction is determined. This process determines the point of steepest slope in the direction specified. Next, the slope at the point of inflection in the specified direction is determined. The same process is followed in each direction from the origin and the steepest slope is determined. All of theses slopes are then compared to determine which is the steepest overall. This value will be used to compute the MDISC statistic. The direction that gives the steepest slope is the same as the direction specified by the MID. The distance from the origin to the point of steepest slope in the direction indicated by the MID, is the distance component of the MID statistic.

The mathematical procedure for determining MDISC has four steps. First, the mathematical expression for the IRS is converted to polar coordinates to simplify the analysis. Second, the second derivative in direction α from the origin is used to determine the point of steepest slope in that direction. Third, the expression for the slope at the point of steepest slope is determined using the first derivative. Finally, the first derivative is taken with respect to α to determine the direction of overall steepest slope. A function of the slope in that direction is proposed as the MDISC statistic.

For all of the IRS's considered so far, this procedure yields a unique value of MDISC that has the same relationship to MID that the a-parameter has to the b-parameter in unidimensional IRT. This relationship will be demonstrated in the next section when the MDISC definition is applied to a particular MIRT model.

Multidimensional Information

Although multidimensional information (MINF) is related to the MDISC in that if an item has a high value of MDISC it will provide a lot of information somewhere in the ability space, the concept is also quite different because it is concerned with the ability of the item to discriminate at each point in the space rather than just at the steepest point of the IRS.

The definition of MINF proposed in this paper is a direct generalization of the unidimensional IRT concept of information. For unidimensional IRT, information at an ability level, θ, is defined as the ratio of the square of the slope of the ICC at ability level θ to the variance of error of the item score at that level of θ. Mathematically, item information is expressed as follows.

$$I_i(\theta) = \frac{\left[\frac{\delta P_i(\theta)}{\delta \theta}\right]^2}{P_i(\theta)\, Q_i(\theta)} \qquad (2)$$

where $\underline{I}_i(\theta)$ is the information from Item i at ability level θ,

$\underline{P}_i(\theta)$ is the probability of a correct response to Item i for a person with ability θ,

and $\underline{Q}_i(\theta) = 1 - \underline{P}_i(\theta)$.

Test information is simply the sum of item information values

$$\underline{I}_T(\theta) = \sum_{i=1}^{n} I_i(\theta) \qquad (3)$$

where $\underline{n}$ is the number of items.

For the multidimensional case, Equations 2 and 3 can still be used, but the slope in the numerator of Equation 2 must be determined in a slightly different way. When an IRS is considered instead of an ICC there are many slopes at any point in the ability space rather than one. Depending on the direction that is taken at the point in the space, the slope will differ. The slope will be much greater if a direction is selected that goes up the surface rather than one that goes across it. One direction may yield a slope of zero while another direction, at the same point in the space, may yield a fairly steep slope. Thus, direction in the space must be considered when determining the information provided by the item. This is the same as specifying how much information is provided about a particular composite of abilities at a point in the ability space.

In order to determine the slope in a particular direction, the mathematical procedure known as the directional derivative is needed. The directional derivative is defined as

$$\nabla_{\alpha} P(\theta) = \frac{\delta \underline{P}(\theta)}{\delta \theta_1} \cos \alpha_1 + \frac{\delta \underline{P}(\theta)}{\delta \theta_2} \cos \alpha_2 + \ . \ . \ . + \frac{\delta \underline{P}(\theta)}{\delta \theta_n} \cos \alpha_n, \quad (4)$$

where α is the vector of angles with the coordinate axes in the θ-space, α_i $(i = 1,\underline{n})$ is an element of the vector, θ is the vector of abilities defining a point in the space, and $\theta_i(i = 1,\underline{n})$ is an element of the vector. Equation 4 gives the slope in direction α at the point θ in the ability space.

When computing the multidimensional information, the directional derivative replaces the derivative in the numerator of Equation 2. When this is done, item information can be determined for any angle, representing different composites of abilities, in the space. Thus, to totally describe the information structure of an item, many information plots are needed, not just one. In principal, an information function can be determined for the infinite number of directions from the origin. In practice, determining the information function for angles at 10° intervals between 0° and 90° from the axes is sufficient to determine where, and for what combination of abilities, an item provides information.

The next section of the paper presents a derivation of MDISC

and MINF for a particular MIRT model. The following section applies the concepts to actual test data from the ACT Mathematics Usage Test.

MDISC and MINF for the M2PL Model

In order to demonstrate the use of MDISC and MINF, a MIRT model is needed that can be used to derive the mathematical expressions for the statistics. Since Reckase (1985) has already developed the MID concept using the multidimensional extension of the two-parameter logistic model (M2PL) and an estimation program is available for the model (McKinley & Reckase, 1983), the M2PL model will be used as an example. However, the concepts can also be applied equally as well to other MIRT models.

The M2PL model is given by

$$P(\underline{x}_{ij} = 1 \mid a_i, \underline{d}_i, \theta_j) = \frac{e^{a_i' \theta_j + d_i}}{1 + e^{a_i' \theta_j + d_i}} \qquad (5)$$

where $\underline{x}_{ij}$ is the score (0, 1) on Item i by Person j,

a_i is the vector of item discrimination parameters,

$\underline{d}_i$ is a scalar parameter that is related to the difficulty of the item

and θ_j is the vector of ability parameters for Person j.

MDISC

The MDISC for an item is a function of the slope at the steepest point in the MID direction for an item. Reckase (1985) derived the MID direction as

$$\cos \alpha_{ik} = \frac{\underline{a}_{ik}}{\left[\sum_{k=1}^{m} \underline{a}_{ik}^2\right]^{1/2}} \tag{6}$$

where α_{ik} is the angle with Axis k for Item i,

$\underline{a}_{ik}$ is the kth element of vector a_i,

and $\underline{m}$ is the number of dimensions in the space.

He also determined that the slope of the IRS at the point of inflection in direction α_i is

$$\text{Slope} = \frac{1}{4} \sum_{k=1}^{m} \underline{a}_{ik} \cos \alpha_{ik} \tag{7}$$

Substituting Equation 6 into Equation 7 yields the slope in the MID direction

$$\text{Slope} = \frac{1}{4} \sum_{k=1}^{m} \underline{a}_{ik} \frac{\underline{a}_{ik}}{\left[\sum_{k=1}^{n} \underline{a}_{ik}^2\right]^{1/2}} = \frac{1}{4}\left[\sum_{k=1}^{m} \underline{a}_{ik}^2\right]^{1/2} . \tag{8}$$

For the unidimensional two-parameter logistic (2PL) model, the

slope at the point of inflection is equal to (1.7) $(1/4)a_i$.

Thus $\left[\sum_{k=1}^{m} a_{ik}^2\right]^{1/2}$ is analogous to the a-parameter in the unidimensional model. Therefore, it is proposed that the MDISC be defined as

$$\text{MDISC} = \left[\sum_{k=1}^{m} a_{ik}^2\right]^{1/2} . \qquad (9)$$

This definition of MDISC has several nice properties. First, if an item measures only dimension ℓ, that is, when $a_{i\ell} > 0$ and $a_{ij} = 0$, $j \neq \ell$, MDISC = $a_{i\ell}$ and the multidimensional discrimination is equal to the unidimensional discrimination parameter, as it should.

Second, when the 2PL model is expressed in the slope-intercept form, the exponent is given by $a_i\theta_j + d_i$ where $d_i = -b_i a_i$. The distance, D_i, in the MID has the same relationship with the intercept term, d_i, of Equation 5 as b_i does with d_i for the 2PL model. That is,

$$d_i = - D_i \cdot \text{MDISC} \qquad (10)$$

since

$$\underline{D}_i = \frac{-\underline{d}_i}{\left[\sum_{k=1}^{m} \underline{a}_{ik}^2\right]^{1/2}} = \frac{-\underline{d}_i}{\text{MDISC}} . \qquad (11)$$

Finally, MDISC is on the same scale as $\underline{a}_{ik}$, namely, four times the slope, so it can be interpreted accordingly. The definition meets all the requirements stated earlier for a generalization of the IRT discrimination parameter.

<u>MINF</u>

In order to compute the MINF for the M2PL model, the directional derivative of the IRS is needed. The directional derivative is given by

$$\begin{aligned}\nabla_{\alpha} \underline{P}_i(\theta) &= \underline{a}_{i1}\underline{P}_i(\theta)\underline{Q}_i(\theta)\cos \alpha_1 + \underline{a}_{i2}\underline{P}_i(\theta)\underline{Q}_i(\theta)\cos \alpha_2 + \ldots \\ &\quad + \underline{a}_{im} \underline{P}_i(\theta)\underline{Q}_i(\theta)\cos \alpha_m , \\ &= \underline{P}_i(\theta)\underline{Q}_i(\theta) \sum_{k=1}^{m} \underline{a}_{ik} \cos \alpha_k . \qquad (12)\end{aligned}$$

This expression can be substituted for the term in the numerator of Equation 2 yielding

$$I_{i\alpha}(\theta) = \frac{\left(\underline{P}_i(\theta)\,\underline{Q}_i(\theta) \sum_{k=1}^{m} \underline{a}_{ik} \cos \alpha_k\right)^2}{\underline{P}_i(\theta)\,Q_i(\theta)}$$

$$= \underline{P}_i(\theta)\,\underline{Q}_i(\theta) \left(\sum_{k=1}^{m} a_{ik} \cos \alpha_k\right)^2 . \qquad (13)$$

From this equation, the information at the point indicated by θ in direction α can be determined. As with the unidimensional definition of information, the item information functions can be summed to obtain a test information function. However, when the test information is computed, the same direction must be used for all of the items.

Example of the Application of MDISC and MINF

In order to demonstrate the use of MDISC and MINF, Form 24B of the ACT Mathematics Usage Test was analyzed to determine estimates of the parameters of the M2PL model. The responses from a systematic sample of 1,000 examinees were used for this purpose. The MAXLOG program (McKinley & Reckase, 1983) was used to estimate the parameters. A two-dimensional solution was obtained for this example so that the results could be represented graphically.

The parameter estimates for the M2PL model, the MID, and the MDISC statistics for the 40 items on the test are presented in Table 1. Of the items on the test, Item 27 has the highest MDISC statistic. This means that of all of the items on the test, this

item was the best at differentiating between examinees in different parts in the θ-space. However, this item discriminates best along a line that is at a 46° angle to the Dimension 1 axis. Along the Dimension 1 axis (at 0° to the axis), the discrimination is only 1.66, which is less than the discrimination for Item 10 along the axis. Thus, the MDISC gives an overall measure of the quality of the item, but it does not indicate that the item is of equal quality in measuring in all directions (i.e., for all weighted composites of abilities).

Insert Table 1 about here

The MDISC statistics for two items can only be directly compared if the items measure in the same direction. For example, Items 29 and 36 can be compared on MDISC because they measure in the same direction. The MDISC statistics cannot be compared for Items 3 and 30 because the directions are quite different. To compare these items, a common dire 'ion α, would have to be selected, and then the discrimination in the α direction would have to be computed using the formula

$$\text{Directional Discrimination} = \sum_{k=1}^{m} a_{ik} \cos \alpha_{ik} \qquad (14)$$

For Items 3 and 30, the directional discriminations in direction

30° from Dimension 1 are 1.54 and 1.19 respectively: at 60° from Dimension 1, they are 1.10 and 1.39 respectively. Thus, depending on the direction, the ordering of the items on discrimination changes. However, Item 3 is more discriminating overall since it has a higher MDISC statistic.

In order to give some further guidance in interpreting the MDISC statistic, the correlation has been computed between it and the biserial correlation between the item and the total score on the test, the a-parameter estimate from the three-parameter logistic model obtained from LOGIST (Wingersky, Barton & Lord, 1982), and the a-parameter estimates from the M2PL model. These correlations are given in Table 2. The MDISC statistic for this set of data was found to be correlated most highly with the r_{bis} statistic. It is interesting that the a-parameter estimate from LOGIST is most highly related to a_2 while r_{bis} is most highly related to a_1. The relationship between MDISC, and a_1 and a_2 is dictated by Equation 9.

Insert Table 2 about here

In order to demonstrate the use of the MINF, the MINF was computed for Item 10 using directions of 0°, 30°, 60°, and 90° from Dimension 1. The results for this item are shown in two different ways in Figures 1 and 2. Figure 1 indicates the amount

of information in direction α by the height of the surface above the θ-plane. The four parts of Figure 1 show the surfaces for each of the four directions. The figures show that the item gives no information about Dimension 2, and that the amount of information provided by the item increases as the angle goes from 90° to 0°. In all cases, the information is greatest along the line, $\theta_1 = -.19$.

Insert Figure 1 about here

This same data is presented in Figure 2 using a representation scheme suggested by David Thissen. At selected points in the θ-space, the information is represented by the length of the line in the direction taken in the space. Lines are give at 10° intervals. Figure 2 shows that Item 10 gives no information about Dimension 2, and progressively more information as the angle goes from 90° to 0° with respect to Dimension 1. Most of the information provided in the four parts of Figure 1 is given in Figure 2.

Insert Figure 2 about here

The information supplied by the entire test is shown by the three surfaces in Figure 3 and the line plots in Figure 4. A

comparison of Figures 3a and 3c shows that the test supplies somewhat more information about θ_1 than θ_2. Figure 3b shows that the most information is provided about an equally weighted composite of θ_1 and θ_2. The same information is given in Figure 4, but the line plot more clearly indicates the regions of the θ-space that are best measured by the test.

Insert Figures 3 and 4 about here

Discussion

The purpose of this paper has been to define conceptually the concepts of multidimensional discrimination and information, to derive the mathematical expressions for the concepts for a particular multidimensional IRT model, and then to apply the concepts to actual test data. Multidimensional discrimination was defined as a function of the slope of the item response surface in the direction specified by the multidimensional difficulty (Reckase, 1985). For the M2PL model, this definition results in a statistic that has the same relationship to multidimensional difficulty that *a* does to *b* for the unidimensional IRT models. This statistic is defined as the square root of the sum of the squared *a*-parameters from the M2PL model.

Multidimensional information was defined by the same mathematical function as unidimensional information, but the directional derivative was substituted for the standard derivative in the numerator of the information expression (Equation 2). The use of the directional derivative results in an information measure that corresponds to a direction in the ability space. Direction, in this case, is an indicator of the composite of abilities that is of interest. Use of multidimensional information makes very clear the dimensions being measured by a test and the regions of the ability space being measured on each dimension.

The ACT Mathematics Usage Test was used to demonstrate the multidimensional discrimination and information. The use of the statistics show that the items on the test vary substantially in their directions of maximum discrimination and that the test tends to measure Dimension 1 or a combination of Dimensions 1 and 2 at a greater level of precision than Dimension 2 alone. The regions of the ability space that are best measured were also clearly indicated.

The multidimensional measures of item quality, item precision, and test precision given in this paper provide another set of tools that can be used to gain a better understanding of the measurement process. Through their use, and the use of multidimensional models, the dominance of unidimensional measures

may finally by broken, leading to a more powerful class of measurement devices.

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Table 1

Item parameters, MID, and MDISC for the items in the ACT Mathematics usage Test, Form 24B

				MID			
Item	a_{i1}	a_{i2}	d_i	α_{i1}	α_{i2}	D_i	MDISC
1	1.81	.86	1.46	25	65	-.73	2.00
2	1.22	.07	.17	4	89	-.14	1.22
3	1.57	.36	.67	13	77	-.42	1.61
4	.71	.53	.44	37	53	-.50	.89
5	.86	.19	.10	12	78	-.11	.88
6	1.72	.18	.44	6	84	-.25	1.73
7	1.86	.29	.38	9	81	-.20	1.88
8	1.33	.34	.69	14	76	-.50	1.37
9	1.19	1.57	.17	53	37	-.09	1.97
10	2.00	.00	.38	0	90	-.19	2.00
11	.87	.00	.03	0	90	-.03	.87

(table continues)

				MID			
Item	a_{i1}	a_{i2}	d_i	α_{i1}	α_{i2}	D_i	MDISC
12	2.00	.98	.91	26	64	-.41	2.23
13	1.00	.89	-.49	42	48	.37	1.34
14	1.22	.14	.54	7	83	-.44	1.23
15	1.27	.47	.29	20	70	-.21	1.35
16	1.35	1.15	-.21	40	50	.12	1.77
17	1.06	.45	.08	23	67	-.07	1.15
18	1.92	.00	.12	0	90	-.06	1.92
19	.96	.22	-.30	13	77	.30	.98
20	1.20	.12	-.28	6	84	.23	1.21
21	1.41	.04	-.21	2	88	.15	1.41
22	1.54	1.79	.02	49	41	-.01	2.36
23	.54	.23	-.69	23	67	1.18	.59
24	1.53	.48	-.83	17	73	.52	1.60
25	.72	.55	-.56	37	53	.62	.91
26	.51	.65	-.49	52	38	.59	.83
27	1.66	1.72	-.38	46	44	.16	2.39

(table continues)

				MID			
Item	a_{i1}	a_{i2}	d_i	α_{i1}	α_{i2}	D_i	MDISC
28	.69	.19	-.68	15	75	95	.72
29	.88	1.12	-.91	52	38	.64	1.42
30	.68	1.21	-1.08	61	29	.78	1.39
31	.24	1.14	-.95	78	12	.82	1.36
32	.51	1.21	-1.00	67	23	.76	1.31
33	.76	.59	-.96	38	52	1.00	.96
34	.01	1.94	-1.92	90	0	.99	1.94
35	.39	1.77	-1.57	78	12	.87	1.81
36	.76	.99	-1.36	52	38	1.09	1.25
37	.49	1.10	-.81	66	24	.67	1.20
38	.29	1.10	-.99	75	15	.87	1.14
39	.48	1.00	-1.56	64	26	1.41	1.11
40	.42	.75	-1.61	61	29	1.87	.86

Table 2

Correlations betwee.. discrimination parameter estimates

Discrimination Estimate	1.	2.	3.	4.	5.
1. MDISC		.46	.78	.62	.46
2. a_{LOGIST}			.14	-.21	.74
3. r_{BIS}				.80	.16
4. a_1					-.34
5. a_2					

Figure Caption

Figure 1. Multidimensional information for Item 10 at angles of 0°, 30°, 60°, and 90° to the θ_1 axis.

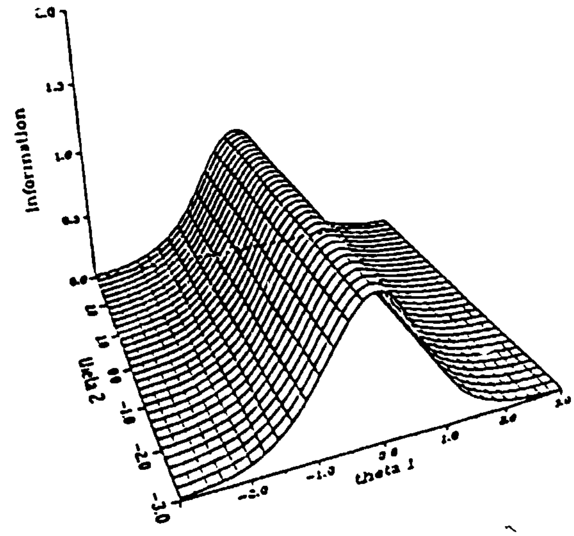
Figure 2. Multidimensional information for Item 10 represented by directional vectors from points in the θ-space.

Figure 3. Multidimensional information for the ACT Mathematics Usage Test at angles of 0°, 45°, and 90° to the θ_1 axis.

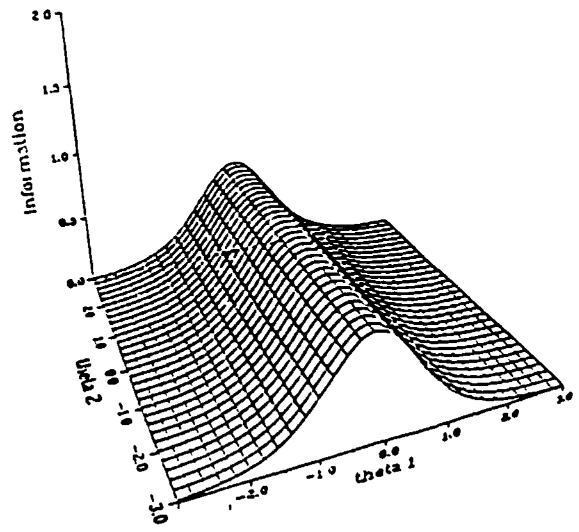
Figure 4. Multidimensional information for the ACT Mathematics Usage Test represented as directional vectors from points in the θ-space.

FIGURE 1

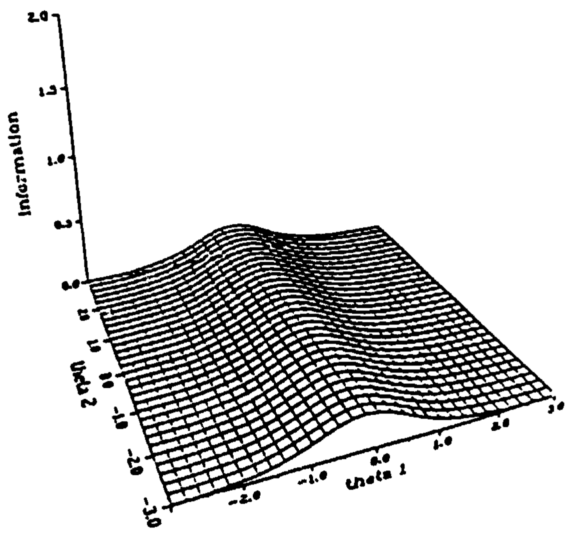
a. 0°



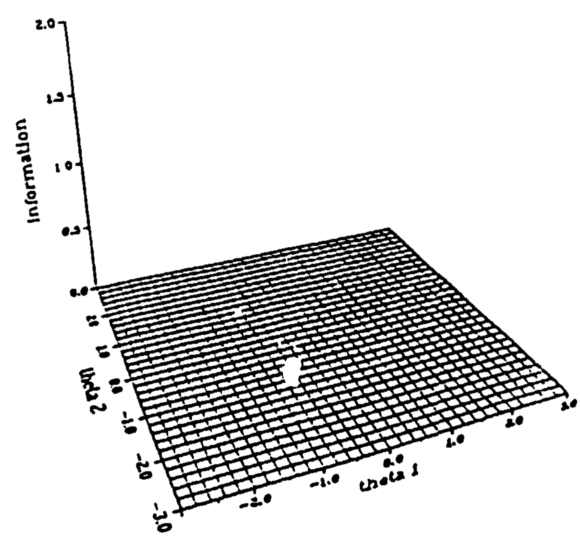
b. 30°



c. 60°



d. 90°



ITEM INFORMATION VECTORS

ITEM 10

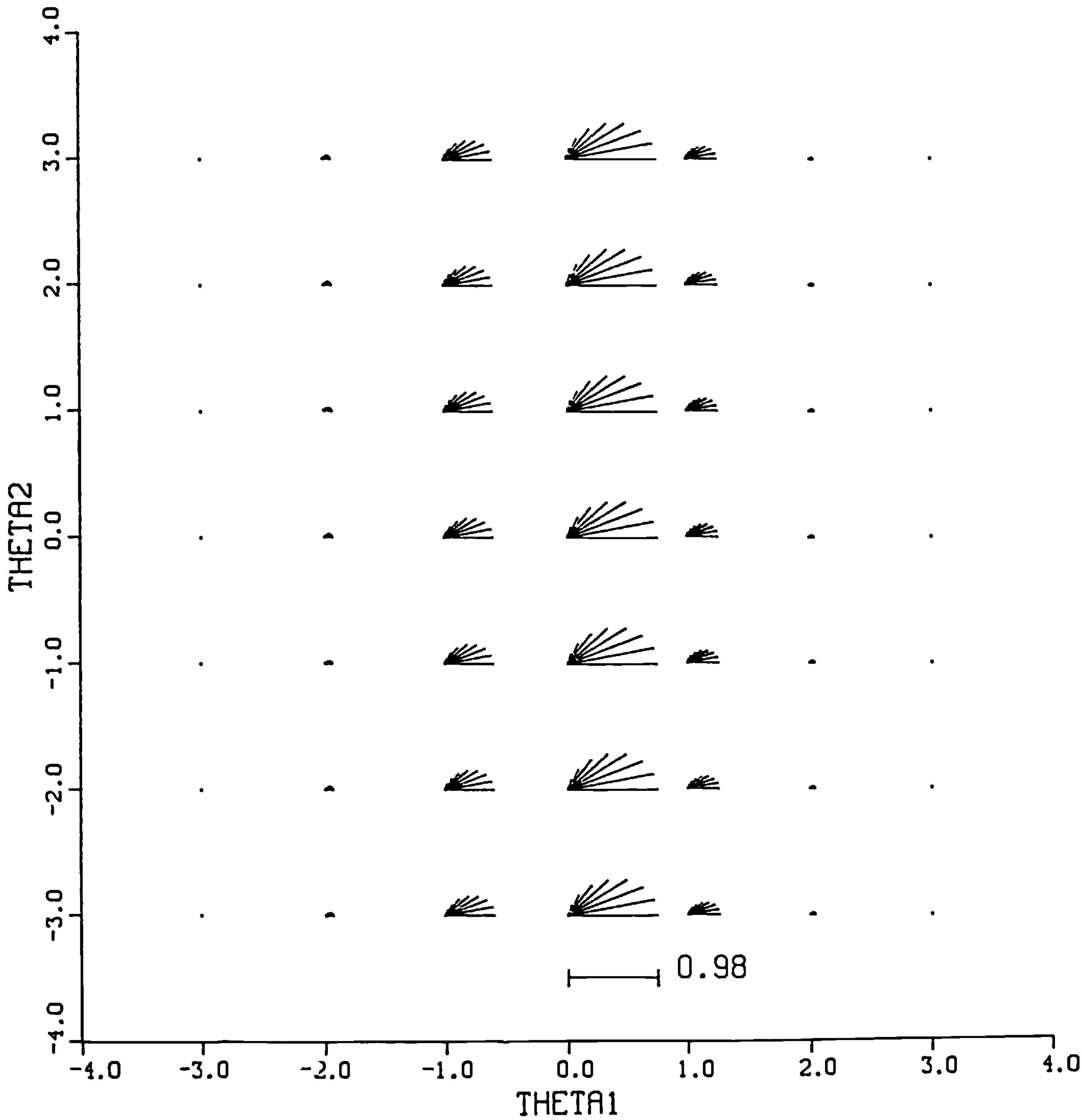


FIGURE 3

a. 0°

b. 45°

c. 90°

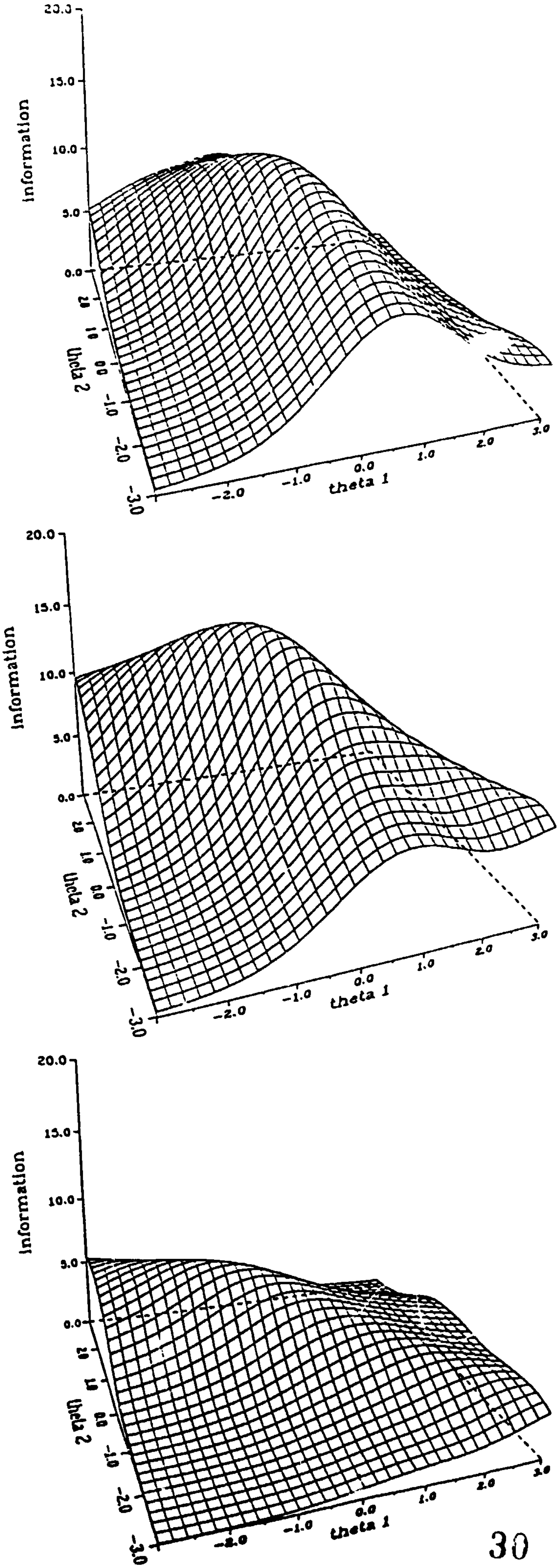


FIGURE 4

TEST INFORMATION VECTORS

FORM 24B